Journal of Population Economics
https://doi.org/10.1007/s00148-021-00824-7

ORIGINAL PAPER

Hyperbolic discounting in an intergenerational model with altruistic parents

Jia Cao[1] · Minghao Li[2]

Received: 7 November 2019 / Accepted: 15 January 2021/Published online: 24 March 2021

Abstract
Hyperbolic utility discounting has emerged as a leading alternative to exponential discounting because it can explain time-inconsistent behaviors. Intuitively, hyperbolic discounting should play a crucial role in intergenerational models characterized by intertemporal trade-offs. In this paper, we incorporate hyperbolic discounting into a dynamic model in which parents are altruistic towards their children. Agents live for four periods and choose levels of consumption, fertility, investment in their children's human capital, and bequests to maximize discounted utility. In the steady state, hyperbolic discounting decreases fertility, increases human capital investment, and shifts consumption towards younger ages. These effects are more pronounced in the time-consistent problem (in which agents cannot commit to a course of action) than in the commitment problem, which can be interpreted as realized and intended actions, respectively. The preference-based discrepancy between intended fertility and realized fertility has important implications for the empirical literature that compares the two.

Keywords Hyperbolic discounting · Fertility · Human capital · Intergenerational model

JEL Classification J11 · I21 · D14 · C61

Responsible editor: Alessandro Cigno

✉ Minghao Li
minghao@nmsu.edu

Jia Cao
caojia@ruc.edu.cn

1 School of Applied Economics, Renmin University of China, Mingde Building 103, Zhongguancun Street, Haidian District, Beijing 100872, China

2 Department of Economics, Applied Statistics & International Business, New Mexico State University, Domenici Hall 212, Las Cruces, NM 88003, USA

1 Introduction

Since Samuelson (1937) introduced exponential utility discounting, economists have widely accepted it as a positive description of intertemporal behavior. Its popularity is mostly due to the mathematical convenience of assuming a constant discounting rate. However, Samuelson himself was well aware of the inconsistency between this model and empirical evidence. He noted that exponential discounting leads to time-consistent behavior, namely that given perfect foresight of external events, people would have no incentive to change their plan when the future comes. As counterexamples, he pointed to the existence of irrevocable trusts and compulsory saving measures, which are tools designed to prevent the future self from changing the current plan.

Hyperbolic discounting has emerged as a leading alternative to exponential discounting (Phelps and Pollak 1968; Laibson 1997; O'Donoghue and Rabin 1999). Instead of assuming a constant discounting rate as in exponential discounting, hyperbolic discounting assumes that the discount rate decreases over time. Empirical evidence strongly favors hyperbolic discounting over exponential discounting: experiments and meta-analyses find lower discount rates over longer time horizons; studies reliably reproduce time-inconsistent behavior in laboratory settings; and modeling exercises show that hyperbolic discounting functions fit data better than exponential discounting (see the review paper by Frederick et al. 2002). Hyperbolic discounting has been used to explain saving (Laibson 1997; Diamond and Köszegi 2003; Cao and Werning 2018), addiction (Gruber and Köszegi 2001), health decisions (Ikeda et al. 2010; van der Pol and Cairns 2002), and so on.

This paper studies the effects of hyperbolic discounting on fertility, human capital investment, and bequests in a dynamic model with overlapping generations and four periods for each generation. Intuitively, time preference should play a critical role in the intergenerational process because many parental decisions involve intertemporal trade-offs. For human capital investment, parents have to exchange current resources for returns to children's human capital later on. For bequests, parents have to save for children's future consumption and investment. And most importantly, the fertility decision sets all the above in motion. However, the effects of hyperbolic discounting in intergenerational decisions have not received sufficient attention.

To our best knowledge, Wrede (2011) and Wigniolle (2013) are the only studies that investigate the effect of hyperbolic discounting on fertility. In Wrede (2011), parents only derive utility from consumption and the number of children; in Wigniolle (2013), parents have no access to a capital market. Furthermore, both papers use Becker and Lewis' (1973) static formulation in which parents care about the number and the quality of children. Under this setting, parents can only affect children through fertility and investments in child quality. Other parent-children interactions, such as bequests, are not permitted because they do not contribute to child quality.

This paper incorporates hyperbolic discounting into Becker and Barro's (1988) reformulated theory of fertility with altruism toward children (we call this the altruism model). Here, "altruism" means that parents' utility function includes children's utility. The altruism model can naturally incorporate complex parent-children interactions since all children's utility determinants also affect parents' utility maximization. Moreover, by using a utility function that iterates through generations, the altruism model is a dynamic model rather than a static one like that introduced by Becker and Lewis (1973) and adopted by Wrede (2011) and Wigniolle (2013).

We extend the existing literature on fertility with hyperbolic discounting by building a richer model that considers fertility, investment in human capital, saving and borrowing, and bequests. Also, with a dynamic model, we can derive steady-state results that match long-term outcomes, such as cross-country differences. Becker and Barro's (1988) model shows that steady-state fertility is determined by a few fundamental factors, including the altruism function and the interest rate. Our extension spells out the effects of another fundamental factor, the hyperbolic discounting rate, on steady-state fertility, human capital investment, bequests, and consumption.

We find that, in the steady state, hyperbolic discounting decreases fertility, increases the investment in human capital, and increases the share of lifetime consumption by the young agent. The effect of hyperbolic discounting on bequests depends on parameter values. Furthermore, hyperbolic discounting has stronger effects when the agent cannot commit to a plan and has to make decisions based on her future self's behavior.

As hyperbolic discounting measurements accumulate, our model's predictions are testable. If true, the change in fertility plan caused by hyperbolic discounting would have important practical implications. For example, hyperbolic discounting can explain lower-than-intended fertility rates for European and US women (Beaujouan and Berghammer 2019). Also, the empirical literature compares women's intended and realized fertility rates to quantify the effects of fertility determinants, such as fertility demand (Pritchett 1994; Günther and Harttgen 2016) and spouses' differential fertility desires (Thomson et al. 1990; Doepke and Tertilt 2018). Preference-based changes in fertility plan, if not controlled for, would be confounded with fertility determinants suggested by these studies.

The paper is organized as follows. Section 2 sets up the general model. Section 3 presents theoretical results without functional form assumptions in subsection 3.1 and those with functional form assumptions in subsection 3.2. Section 4 concludes and discusses the links between this study and the empirical literature.

2 The model

A family is assumed to include only one parent and her children. An agent lives for four periods: childhood, youth, middle-age, and old-age. In childhood (period zero), the agent receives human capital investment from her parent but does not make any decisions. Since children are not autonomous in period zero, we follow Galor and Zeira (1993) and assume that children in period zero have no consumption.

When the agent is young (period one), she receives a bequest from her parent and makes consumption decisions. At middle-age (period two), she makes decisions on consumption, fertility, and human capital investment in her children. In this period, her children are born and start childhood. When old (period three), the agent retires (stops making an income) and makes consumption and bequest decisions. In all periods, income not used in consumption or expenditures on children becomes saving. All children are assumed to be identical. We also assume that children's discounted lifetime utility in period one (period three for the parent) enters the parent's utility function.

In our model, the decisions of fertility and human capital investment are both made in period two. This assumption cannot be relaxed easily in our model: if the parent decides on fertility in period one, then the agent cannot change her fertility plan and hyperbolic discounting would not manifest in fertility behaviors.[1] If human capital investment were to be postponed to parent's period three, it would be indistinguishable from the bequest. To solve the above dilemma, one could use a five-period model, which would be difficult to solve and interpret. However, we will discuss the likely results from a model with human capital investment after fertility decision.

The agent's problem is to choose consumption levels in periods one to three, fertility and the human capital investment in period two, and the bequest in period three to maximize her discounted lifetime utility. The agent's external income comes from wage labor. The wage in period one is $h(e)$ where e is the human capital, with $h'(e) > 0$ *and* $h''(e) < 0$. The wage in period two is $(1 + g)h(e)$, where g is the exogenous wage growth rate, reflecting human capital growth from work experience.

By assuming exogenous wage and wage growth, we abstract from macroeconomic feedback and focus the analysis on the family. At any given period, the agent's utility (value function) is the sum of her current utility from consumption, her discounted utilities from consumption in future periods, and the discounted utility from children. The value function of an agent in period t is denoted by $V_t(e, n, a_t)$ where e, n, and a_t are the agent's received human capital investment, the number of children, and the wealth at the beginning of period t. In period one, an agent's wealth equals her received bequest (b). The corresponding variables for the next generation are e', a_t', and b'. We assume that agents follow hyperbolic utility discounting with the quasi-hyperbolic specification (Laibson 1997). At time t, the discount rate for utility at time t' is $\beta\delta^{t'-t}$ with $\beta\,(0 \le \beta \le 1)$ being the hyperbolic parameter and δ $(0 \le \delta \le 1)$ being the time discounting rate. Exponential discounting is a special case of hyperbolic discounting when $\beta = 1$. The following Bellman equations represent discounted utility in the three periods:

Period one:

$$V_1(e,b) = max_{c_1,c_2,c_3,n,e',b'}\{u(c_1) + \beta\delta u(c_2) + \beta\delta^2 u(c_3) + \beta\delta^2\Phi(n)V_1(e',b')\} \quad (1)$$

Period two:

$$V_2(e,a_2) = max_{c_2,c_3,n,e',b'}\{u(c_2) + \beta\delta u(c_3) + \beta\delta\Phi(n)V_1(e',b')\} \quad (2)$$

Period three:

$$V_3\left(e',n,a_3\right) = max_{c_3,b'}\{u(c_3) + \Phi(n)V_1(e',b')\} \quad (3)$$

where $u(c) > 0, \forall\, c > 0$.

[1] It is relatively easier to place the education decision after the fertility decision is a static model, such as that in Nakagawa et al. (forthcoming).

The altruism function $\Phi(n)$ satisfies the following conditions:

$$\Phi(0) = 0, \Phi'(n) > 0, \Phi''(n) < 0, \lim_{n\to 0}\Phi'(n) = +\infty, \lim_{n\to +\infty}\Phi'(n) = 0.$$

Children enter parent's utility function as the product of the lifetime utility for each child $V_1(e', b')$ and an altruism function $\Phi(n)$ for n children. The assumptions on the properties of $\Phi(n)$ are satisfied in the Beckerian model (Becker and Barro 1988) where $\Phi(n) = \alpha n^{1-\epsilon}$, $\alpha > 0$, and $0 < \epsilon < 1$. The assumptions of $u(c) > 0$ and $\lim_{n\to 0}\Phi'(n) = +\infty$ together exclude childlessness as an optimal solution: the utility from children is always positive, and the marginal utility of having children is positive infinity when the parent is childless.

Suppose an agent can commit to her decisions or not foresee her future self's time preference. In that case, she will make decisions for all periods to maximize her discounted utility in the first period (Eq. (1)). We refer to this optimization problem as the commitment problem. With hyperbolic discounting, the agent will change her first-period optimal plans without effective commitment tools. Assuming the existence of a credit market, the budget constraint in the commitment problem is:

$$c_1 + \frac{c_2}{R} + \frac{c_3}{R^2} = h(e) + b + \frac{(1+g)h(e)(1-nx) - ne'}{R} - \frac{nb'}{R^2} \tag{4}$$

The consumption in period t is c_t. The gross interest rate is R. The expenditures on children have three components. The first component is the time cost $(1+g)h(e)nx$, which only depends on the number of children, and the portion of a parent's time used to raise one child x $(0 \le x \le 1)$. We assume that this cost is fixed for each child, making x an exogenous parameter. The second component ne' is the monetary cost for acquiring children's human capital. The unit of human capital is chosen such that the price for human capital is one. Consequently, a child's human capital equals the human capital investment she receives from her parent. The third component nb' is the bequest.

Without commitment tools, a sophisticated agent would make decisions such that her future self will not change the plan. We call this optimization problem the time-consistent problem. Assuming the existence of a credit market, the budget constrains in the time-consistent problem are:

$$\text{Period one}: c_1 = h(e) + b - \frac{a_2}{R}$$

$$\text{Period two}: c_2 = a_2 + (1+g)h(e)(1-nx) - ne' - \frac{a_3}{R}$$

$$\text{Period three}: c_3 = a_3 - nb'$$

In the first two periods, the agent will maximize her utility in each period (Eqs. (1) and (2), respectively) while treating her decisions in the following period as given. In each period, the budget constraint equates parent's consumption (c_1, c_2, and c_3 for the three periods respectively) with the resources available ($h(e) + b$, $a_2 + (1+g)h(e)$, and a_3) minus expenses on children (0, $(1+g)h(e)nx + ne'$, and nb') and savings ($\frac{a_2}{R}$, $\frac{a_3}{R}$, and 0).

3 Results

In this section, we first present results that can be derived without specific functional form assumptions in subsection 3.1 and then present more additional results with functional form assumptions in subsection 3.2.

3.1 Model solution without functional form assumptions

The steady-state solutions and value functions are marked by superscripts c and t respectively in the commitment problem and the time-consistent problem. All proofs are in the Appendix.

Proposition 1. *In the commitment problem, the steady-state fertility is determined by the following equation:*

$$\beta\delta^2 R^2 \frac{\Phi(n^c)}{n^c} = 1 \tag{5}$$

Proposition 2. *In the time-consistent problem, the steady-state fertility is determined by the following equation:*

$$(\beta\delta R)^2 \frac{\Phi(n^t)}{n^t} = 1 \tag{6}$$

Propositions 1 and 2 show that, in both the commitment and time-consistent problems, three fundamental factors determine the steady-state fertility: the hyperbolic discounting rate β, the time discounting rate δ, and the interest rate R. Intuitively, the steady-state fertility decreases with β and δ because the future utility from children will be more heavily discounted if β and δ are smaller. Since the parent has to save for expenditures on children, the higher the interest rate, the lower the real cost of children. Fertility in the steady state is independent of wages and wage growth rate, echoing Becker and Barro (1988). The absence of wages and wage growth rate in the steady-state fertility is because time cost, which is proportional to wages, is the only fixed cost for each child. Therefore, the price effect of wages and wage growth on fertility balances out the income effect (see Jones et al. (2010) for a discussion of this result in the Beckerian model without hyperbolic discounting).

The altruism function $\Phi(n)$ increases fertility in both problems, as expected. However, fertility does not depend on the utility function for consumption ($u(c)$) in either of the two problems. In the steady state of a dynamic problem, $u(c)$ would appear in both parent's consumption utility and the utility from children. A change in $u(c)$ has competing effects: for example, an increase in the utility from consumption would increase consumption and decrease fertility; at the same time, the utility from children would also increase, which leads to higher fertility. Propositions 1 and 2 show that the two effects cancel out in the steady state of this problem.

Proposition 3. *In both the commitment problem and time-consistent problem, the steady-state human capital is determined by the following equation:*

$$h'(e) = \frac{R^2}{R + (1+g)(1-nx)} \tag{7}$$

Since $h'(e)$ decreases with e, factors that increase the RHS of the above equation decrease e. Proposition 3 shows that the human capital investment has a negative relationship with the number of children, reflecting the quantity-quality trade-off. As a result, the steady-state human capital increases with the hyperbolic discounting parameter β and the time discount parameter δ, and decreases with the interest rate R, all of which act through n. Also, human capital investment increases with the return to human capital $h'(e)$ and wage growth rate g, as expected.

When $\beta = 1$, the commitment problem and time-consistent problem degenerate to the same exponential discounting problem. Let n^e and e^e denote the steady-state fertility and human capital in the exponential discounting problem. The following proposition describes the relationships between steady-state fertility and human capital in the three problems.

Proposition 4. *In the steady state,* $n^e > n^c > n^t$ *and* $e^e < e^c < e^t$*.*

Proposition 4 states that hyperbolic discounting reduces fertility and increases human capital investment in both the commitment problem and time-consistent problem. The results on fertility are similar to those from Wrede (2011) when children are investment goods. However, Wrede (2011) only models the number of children. Using a static three-period model, Wigniolle (2013), using a model without a capital market, also shows that hyperbolic discounting reduces fertility in interior solutions. Different from these two studies, our model is dynamic and provides steady-state results.

Proposition 4 also tells us that, in the steady state, fertility is lower in the time-consistent problem than in the commitment problem. The difference is positively related to β. In the time-consistent problem, the parent in period one anticipates herself in period two to discount children's utility, which is in period three, by an additional factor of β. Therefore, she has to choose a lower fertility to maintain time consistency. Since e is inversely related to n, the inequalities are reversed for human capital. A sophisticated parent would prevent time-inconsistent behavior by lowering fertility level; at the same time, she would increase human capital investment accordingly.

3.2 Model solution with functional form assumptions

To explore the effects of hyperbolic discounting on consumption and bequests and obtain closed-form solutions for fertility and human capital in the steady state, we assume the functional forms for utility and wage functions to be:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \Phi(n) = \frac{n^{1-\epsilon}}{1-\epsilon}, h(e) = e^{\sigma}, with\ 0 < \gamma, \epsilon, \sigma < 1, \gamma \neq \epsilon, R \neq \left(\frac{1-\epsilon}{\beta\delta}\right)^{\frac{1}{1-\epsilon}}, and\ \ R \neq \left(\frac{1-\epsilon}{\beta\delta^2}\right)^{\frac{1}{2-2\epsilon}}.$$

The last three inequalities prevent dividing by zero.

Proposition 5. *With the functional form assumptions above, the steady-state fertility rates in the two problems have the following relationships*:

$$n^t = n^c \beta^{\frac{1}{\epsilon}}, or\ \log(n^t) = \log(n^c) + \frac{1}{\epsilon}\log(\beta).$$

The relationships above can be matched to empirical data if we assume that n^c and n^t represent intended and realized fertility, respectively. The first equation indicates that the ratio between realized and intended fertility identifies $\beta^{\frac{1}{\epsilon}}$. If β is available, one can use regression to estimate ϵ from the second equation and vice versa.

Proposition 6. *With the above utility, altruism, and wage functions, the steady-state consumption levels in three periods of the commitment problem are* $c_1^c = \alpha_1 I^c$, $c_2^c = \alpha_2 I^c$, *and* $c_3^c = \alpha_3 I^c$ *where*

$$\alpha_1 = 1/\left[1 + (\beta\delta R)^{\frac{1}{\gamma}} R^{-1} + \left(\beta\delta^2 R^2\right)^{\frac{1}{\gamma}} R^{-2}\right]$$
$$\alpha_2 = 1/\left[R^{-1} + (\beta\delta R)^{-\frac{1}{\gamma}} + (\delta R)^{\frac{1}{\gamma}} R^{-2}\right]$$
$$\alpha_3 = 1/\left[R^{-2} + \left(\beta\delta^2 R^2\right)^{-\frac{1}{\gamma}} + (\delta R)^{-\frac{1}{\gamma}} R^{-1}\right]$$
$$I^c = \frac{1-\gamma}{\gamma-\epsilon}\left[(1+g)(e^c)^{\sigma} xR + e^c R - (e^c)^{\sigma} - \frac{(1+g)(e^c)^{\sigma}}{R}\right]$$

with α_1, α_2, *and* α_3 *being the shares of discounted lifetime consumption* I^c *consumed in each period in the commitment problem.*

Proposition 7. *With the above utility, altruism, and wage functions, the steady-state consumption levels in three periods of the time-consistent problem are* $c_1^t = \mu_1 I^t$, $c_2^t = \mu_2 I^t$, $c_3^t = \mu_3 I^t$ *where*

$$\mu_1 = 1/\left[1 + (\beta\delta R)^{\frac{1}{\gamma}} R^{-1} + (\beta\delta R)^{\frac{2}{\gamma}} R^{-2}\right]$$
$$\mu_2 = 1/\left[R^{-1} + (\beta\delta R)^{-\frac{1}{\gamma}} + (\beta\delta R)^{\frac{1}{\gamma}} R^{-2}\right]$$
$$\mu_3 = 1/\left[R^{-2} + (\beta\delta R)^{-\frac{1}{\gamma}} R^{-1} + (\beta\delta R)^{-\frac{2}{\gamma}}\right]$$
$$I^t = \frac{1-\gamma}{\gamma-\epsilon}\left[(1+g)(e^t)^{\sigma} xR + e^t R - (e^t)^{\sigma} - \frac{(1+g)(e^t)^{\sigma}}{R}\right]$$

with μ_1, μ_2, *and* μ_3 *being the share of discounted lifetime consumption* I^t *consumed in each period in the time-consistent problem.*

Propositions 6 and 7 indicate that consumption decisions are made in two steps. First, lifetime consumption is determined jointly with decisions with expenditures on children because the expressions for I^c and I^t contain endogenous human capital. The effect of hyperbolic discounting on lifetime consumption depends on parameter values. Second, lifetime consumption is distributed to the three periods based on β, δ, R, and γ, independent from children-related decisions or altruism towards children.

The forms of α's and μ's indicate that in both problems, the steady-state consumption share in period one is higher due to hyperbolic discounting and the consumption share in the last period is lower. Furthermore, with hyperbolic discounting, the consumption share in period 2 is lower in the commitment problem; it is also lower in the time-consistent problem if β is sufficiently small ($\beta < \delta^{-1}R^{-1+\gamma}$). This condition can be satisfied if $\delta R \approx 1$. As expected, the extra hyperbolic discounting shifts consumption earlier. This shift is more pronounced in the time-consistent problem following the same logic that steady-state fertility is lower in the time-consistent problem. The effect of hyperbolic discounting on bequests depends on parameter values. The proof and solutions for fertility, bequests, and human capital are in the Appendix.

4 Conclusion and discussion

This paper incorporates hyperbolic discounting into a dynamic model with altruistic parents, overlapping generations, and four life-periods for each generation. In our model, the agent receives human capital investment in period zero, starts to work and save in period one, makes fertility and human capital investment decisions in period two (period zero for the child), starts to enjoy utility from children and leaves the bequest in period three. Parent's expenditure on children includes a fixed time cost per child, human capital investment, and bequests. The agent can freely borrow and save at the market interest rate and discounts future utilities using quasi-hyperbolic discounting. Steady-state results are derived for fertility, human capital investment, bequests, and consumption. We separately solve the commitment problem in which agents make all the decisions in period one and the time-consistent problem in which agents make decisions in each period.

We find that the effects of hyperbolic discounting are qualitatively similar in the commitment problem and the time-consistent problem. Because hyperbolic discounting contributes to overall discounting, it diminishes the discounted utility from children and decreases fertility. Through the quantity-quality trade-off, hyperbolic discounting leads to higher investment in human capital. With hyperbolic discounting, the agent consumes a larger share of total lifetime consumption when she is young. The effect of hyperbolic discounting on bequests depends on the values of parameters.

The above theoretical results can be empirically tested. Wrede (2011), who derives similar fertility results using a different model, tests the hypothesis with survey data. Using time-inconsistent saving behavior as a proxy for hyperbolic discounting, he finds the expected negative correlation between fertility and hyperbolic discounting. As hyperbolic discounting measurements accumulate, the next step for future research is to correlate direct measures of hyperbolic discounting with various outcomes. This study's predictions can be tested using aggregated data, such as the national hyperbolic discounting measures constructed by Wang et al. (2016); they can also be tested at the individual-level using custom surveys. The steady-state results from this study match well with the long-term outcomes observed in cross-sectional comparisons.

While qualitatively similar, hyperbolic discounting effects are more pronounced in the time-consistent problem than the commitment problem. Specifically, the agent in the time-consistent problem has fewer children, invests more in human capital, and consumes a larger share of lifetime consumption earlier in life than the agent in the commitment problem. The outcomes in the commitment and time-consistent problems can be interpreted as intended and realized outcomes, respectively. Our results show that realized fertility is lower than fertility intention due to hyperbolic discounting time preference.

This preference-based discrepancy between intended and realized fertility has important empirical implications. First, in developed countries, realized fertility is lower than fertility intention (Adsera 2006; Beaujouan and Berghammer 2019), which has been attributed to various psychological and social factors (Bachrach and Morgan 2013). Without ruling out other explanations, this study offers a parsimonious alternative theory based on basic human behavior. Second, the empirical literature relies on comparing intended and realized fertility to quantify the effects of fertility determinants. For example, the influential study by Pritchett (1994) and the follow-up study by Günther and Harttgen (2016) argue for the dominant role of fertility intention, as opposed to contraception, in determining fertility. Their study's underlying assumption is that intended fertility and realized fertility should have a one-to-one relationship if realized fertility is entirely driven by fertility demand. This assumption is not valid with hyperbolic discounting. Another strand of the empirical literature (for example, Thomson et al. 1990; Doepke and Tertilt 2018) studies how wives' and husbands' fertility intentions separately affect realized fertility. Lower-than-intended fertility due to hyperbolic discounting would bias the results such that the spouse with lower fertility desire appears to have more influence.

To keep the model tractable, we study a four-period model with fertility and human capital investment decisions happening in the same period. Based on our results, we can speculate the likely results from a five-period model with human capital investment decisions after the fertility decision. In the commitment problem, hyperbolic discounting happens once from the initial period to the next. In the time-consistent problem, hyperbolic discounting happens once more when the agent takes action. In both cases, the number of times that hyperbolic discounting happens is not affected by the delay in human capital investment. Therefore, the delay in human capital investment likely will not qualitatively change the effects of hyperbolic discounting on fertility and human capital investment.

Appendix

Appendix 1. The proof of Proposition 1

Proof. Let λ be the Lagrangian multiplier associated with the budget constraint. Optimal consumption choices satisfy

$$\lambda = u^{'}(c_1) = \beta\delta R u^{'}(c_2) = \beta\delta^2 R^2 u^{'}(c_3) \tag{8}$$

The optimal human capital investment, fertility, and bequest conditions read

$$\frac{\lambda n}{R} = \beta\delta^2 \Phi(n) \frac{\partial V_1^c(e', b')}{\partial e'}$$
$$\lambda\left[\frac{(1+g)h(e)x + e'}{R} + \frac{b'}{R^2}\right] = \beta\delta^2 \Phi'(n) V_1^c(e', b')$$
$$\frac{\lambda n}{R^2} = \beta\delta^2 \Phi(n) \frac{\partial V_1^c(e', b')}{\partial b'}$$

From the Envelope Theorem (ET thereafter), we have

$$\frac{\partial \mathrm{V}_{11}^{\mathrm{c}}(e,b)}{\partial e} = \lambda\left[1 + \frac{(1+g)(1-nx)}{R}\right] h'(e)$$
$$\frac{\partial \mathrm{V}_1^{\mathrm{c}}(e,b)}{\partial b} = \lambda$$

In the steady state, the equation above and the bequest condition decide fertility

$$\beta\delta^2 R^2 \frac{\Phi(n^c)}{n^c} = 1$$

Appendix 2. The proof of Proposition 2

We use backward induction to solve the time-consistent problem: first, Self 3 chooses c_3 and b' given e', a_3 and n; then, Self 2 chooses c_2, n, $e^{'}$, and a_3 given a_2; at last, Self 1 chooses c_1 and a_2 given e and b. The three problems are described as follows.

Self 3 solves the following maximization problem:

$$V_3^t\left(e^{'}, a_3, n\right) = \max_{c_3, b'} \left\{u(c_3) + \Phi(n) V_1^t\left(e^{'}, b'\right)\right\}$$

subject to

$$c_3 = a_3 - nb'$$

The solutions to Self 1's problem are denoted by $c_3 = c_3^t\left(e^{'}, a_3, n\right), b = b^t\left(e^{'}, a_3, n\right)$.

Self 2 solves the following maximization problem:

$$V_2^t(e, a_2) = \max_{c_2, a_3, n, e'} \left\{u(c_2) + \beta\delta V_3^t\left(e^{'}, a_3, n\right)\right\}$$

subject to

$$c_2 = a_2 + (1+g)h(e)(1-nx)-ne' - \frac{a_3}{R}$$

The solutions to Self 2's problem are denoted by $c_2 = c_2^t(e, a_2), a_3 = a_3^t(e, a_2)$, $n = n^t(e, a_2)$.

Self 1 solves the following maximization problem:

$$V_1^t(e, b) = \max_{c_1, a_2}\left\{u(c_1) + \beta\delta V_2^t(e, a_2)\right\}$$

subject to

$$c_1 = h(e) + b - \frac{a_2}{R}$$

The solution to Self 1's problem is denoted by $c_1 = c_1^t(e, b), a_2 = a_2^t(e, b)$.

Self 3's first-order condition (FOC thereafter) with respect to bequests is

$$u'(c_3)n = \Phi(n)\frac{\partial V_1^t(e', b')}{\partial b'}$$

Self 3's ET conditions are

$$\frac{\partial V_3^t\left(e', a_3, n\right)}{\partial e'} = \Phi(n)\frac{\partial V_1^t(e', b')}{\partial e'}$$
$$\frac{\partial V_3^t\left(e', a_3, n\right)}{\partial a_3} = u'(c_3)$$
$$\frac{\partial V_3^t\left(e', a_3, n\right)}{\partial n} = -u'(c_3)b' + \Phi'(n)V_1^t(e', b')$$

Self 2's FOCs with respect to e', a_3, n are

$$u'(c_2)n = \beta\delta\frac{\partial V_3^t\left(e', a_3, n\right)}{\partial e'}$$
$$\frac{u'(c_2)}{R} = \beta\delta\frac{\partial V_3^t\left(e', a_3, n\right)}{\partial a_3}$$
$$u'(c_2)(1+g)h(e)x = \frac{\partial V_3^t\left(e', a_3, n\right)}{\partial n}$$

Self 2's ET conditions are

$$\frac{\partial V_2^t(e, a_2)}{\partial e} = u'(c_2)(1+g)h'(e)(1-nx)$$
$$\frac{\partial V_2^t(e, a_2)}{\partial a_2} = u'(c_2)$$

Self 1's FOC with respect to a_2 and ET conditions read

$$\frac{u'(c_1)}{R} = \beta\delta \frac{\partial V_2^t(e, a_2)}{\partial a_2}$$
$$\frac{\partial V_1^t(e, b)}{\partial e} = u'(c_1)h'(e) + \beta\delta \frac{\partial V_2^t(e, a_2)}{\partial e}$$
$$\frac{\partial V_1^t(e, b)}{\partial b} = u'(c_1)$$

From the FOCs and ET conditions with respect to a_2 and a_3, we have the following equation to describe consumption:

$$u'(c_1) = \beta\delta R u'(c_2) = (\beta\delta R)^2 u'(c_3) \tag{9}$$

In the steady state, we also have

$$u'(c_3)n = \Phi(n)\frac{\partial V_1^t(e, b)}{\partial b} = \Phi(n)u'(c_1)$$

Combining the two equations above to eliminate consumption, we have

$$(\beta\delta R)^2 \frac{\Phi(n^t)}{n^t} = 1$$

Appendix 3. The proof of Proposition 3

In the commitment problem, combining the FOC and ET conditions with respect to human capital from A1 and using the fact that $V_1^c(e, b) = V_1^c(e', b')$ in the steady state, we have

$$\beta\delta^2\Phi(n)\left[1 + \frac{(1+g)(1-nx)}{R^2}\right]h'(e) = \frac{n}{R}$$

Rearranging the equation and using the result of Proposition 1, we have

$$\left[\frac{1}{R} + \frac{(1+g)(1-n^c x)}{R^2}\right]h'(e^c) = 1$$

In the time-consistent problem, combining Self 2's FOC and all three ET conditions with respect to education,

$$u'(c_2) = \beta\delta\Phi(n)\left[u'(c_1)h'(e) + \beta\delta u'(c_2)(1+g)h'(e)(1-nx)\right]$$

Plugging Eq. (8) into the equation above to eliminate consumption and using the result of Proposition 2, we have

$$\left[\frac{1}{R}+\frac{(1+g)(1-n^t x)}{R^2}\right]h'(e^t)=1$$

Appendix 4. The proof of Proposition 4

In both the commitment problem and the time-consistent problem, a reduction in β will lead to an increase in $\Phi(n)/n$, and therefore will reduce n because $\Phi(n)$ is strictly increasing and concave. Hyperbolic discounting means that $\beta < 1$. So, the existence of hyperbolic discounting will reduce fertility in the steady states of both problems. From Eq. (7), we can see a negative relationship between fertility and human capital. Consequently, the existence of hyperbolic discounting will increase the steady-state human capital in both problems.

Dividing Eq. (6) by Eq. (5), we have

$$\beta\frac{\Phi(n^t)}{n^t}=\frac{\Phi(n^c)}{n^c}\Rightarrow\frac{\Phi(n^t)}{n^t}>\frac{\Phi(n^c)}{n^c}\Rightarrow n^t<n^c$$

Combining Eq. (7) with the inequality above, we have

$$n^t<n^c\Rightarrow 1-n^t>1-n^c\Rightarrow h'(e^t)<h'(e^c)\Rightarrow e^t>e^c$$

When $\beta = 1$, we have an exponential discounting problem with solutions n^e and e^e satisfying $\delta^2 R^2\frac{\Phi(n^e)}{n^e}=1$ and $\left[\frac{1}{R}+(1+g)\frac{(1-n^e x)}{R^2}\right]h'(e^e)=1$. So, we have

$$\beta\frac{\Phi(n^c)}{n^c}=\frac{\Phi(n^e)}{n^e}\Rightarrow\frac{\Phi(n^c)}{n^c}>\frac{\Phi(n^e)}{n^e}\Rightarrow n^c<n^e$$

Similarly, we have $e^c > e^e$.

Appendix 5. The proof of Proposition 5

From Appendix 4, we have $\beta\frac{\Phi(n^t)}{n^t}=\frac{\Phi(n^c)}{n^c}$. Plugging $\Phi(n)=\frac{n^{1-\epsilon}}{1-\epsilon}$ into the equation, we have $n^t=n^c\beta^{\frac{1}{\epsilon}}$ which can be used to estimate $\beta^{\frac{1}{\epsilon}}$ as a whole coefficient with n^t and n^c. Taking logs on both sides, we have $\log(n^t)=\log(n^c)+\frac{1}{\epsilon}\log(\beta)$ which can be used to estimate β with n^t, n^c, and ϵ, or be used to estimate ϵ with n^t, n^c, and β.

Appendix 6. The proof of Proposition 6

Define $\theta(c)=\frac{u'(c)c}{u(c)}, \theta(n)=\frac{\Phi'(n)n}{\Phi(n)}$. From Eq. (8) and the budget constraint (4), the steady-state consumption in three periods in the commitment problem can be expressed as

$$c_1^c = \alpha_1 I^c, c_2^c = \alpha_2 I^c, c_3^c = \alpha_3 I^c \tag{10}$$

where α_1, α_2, α_3 are defined as in Proposition 6. $I^c = c_1^c + \frac{c_2^c}{R} + \frac{c_3^c}{R^2}$. We need to show the specific form of I^c. For simplicity, I will not show the superscript "c" until the end of the proof. Substituting the three equations above into Eq. (1), the maximization problem becomes

$$\begin{aligned} V_1(e,b) &= max_{c_1,n,e',b'}\left\{(\alpha_1)^{-\gamma}\frac{I^{1-\gamma}}{1-\gamma} + \beta\delta^2\Phi(n)V_1(e',b')\right\} \\ &= max_{c_1,n,e',b'}\left\{\frac{u(c_1)}{\alpha_1} + \beta\delta^2\Phi(n)V_1(e',b')\right\} \end{aligned} \tag{11}$$

We have three new FOCs, which are given as follows:

$$\begin{gathered} \frac{\partial V_1(e,b)}{\partial b} = u'(c_1) \\ u'(c_1)\frac{n}{R^2} = \beta\delta^2\Phi(n)\frac{\partial V_1(e',b')}{\partial b'} \\ u'(c_1)\left(\frac{(1+g)h(e)x + e'}{R} + \frac{b'}{R^2}\right) = \beta\delta^2\Phi'(n)V_1(e',b') \end{gathered}$$

From the first two FOCs, we have

$$u'(c_1) = \beta\delta^2 R^2\frac{\Phi(n)}{n}u'\left(c_1'\right)$$

From the third FOC, we have

$$\Phi(n)V_1(e',b') = u'(c_1)\left(\frac{(1+g)h(e)x + e'}{R} + \frac{b'}{R^2}\right)\frac{\Phi(n)}{\beta\delta^2\Phi'(n)}$$

Rearranging the equation above with the definition of $\theta(c)$ and $\theta(n)$,

$$\Phi(n)V_1(e',b') = \frac{u(c_1)}{c_1}\left(\frac{(1+g)h(e)nx + ne'}{R} + \frac{nb'}{R^2}\right)\frac{\theta(c_1)}{\beta\delta^2\theta(n)}$$

Using Eq. (10) to rewrite the budget constraint (4),

$$\frac{c_1}{\alpha_1} = c_1 + \frac{c_2}{R} + \frac{c_3}{R^2} = \left[h(e) + b + \frac{(1+g)h(e)}{R}\right] - \left[\frac{(1+g)h(e)nx + ne'}{R} + \frac{nb'}{R^2}\right] \tag{12}$$

Combining the two equations above, we have

$$\Phi(n)V_1(e',b') = \frac{u(c_1)}{c_1}\left(h(e) + b + \frac{(1+g)h(e)}{R} - \frac{c_1}{\alpha_1}\right)\frac{\theta(c_1)}{\beta\delta^2\theta(n)} \tag{13}$$

Rewriting the third FOC to the previous period and using the fact that it's in steady state, we have

$$V_1(e,b) = \frac{u'(c_1)}{\beta\delta^2\Phi'(n)}\left(\frac{(1+g)h(e)x+e}{R}+\frac{b}{R^2}\right) = \frac{R^2u'(c_1)}{\theta(n)}\left(\frac{(1+g)h(e)x+e}{R}+\frac{b}{R^2}\right) \tag{14}$$

Rewriting the equation above, we have

$$V_1(e,b) = \frac{\theta(c_1)}{\theta(n)}[(1+g)h(e)x+eR+b]\frac{u(c_1)}{c_1} \tag{15}$$

Substituting (14) and (15) into (11), we can get the expression for c_1 in the steady state,

$$c_1^c = \alpha_1\frac{\theta(c_1^c)}{\theta(n^c)-\theta(c_1^c)}\left[(1+g)h(e^c)xR+e^cR-h(e^c)-\frac{(1+g)h(e^c)}{R}\right] \tag{16}$$

Combining (12) and (16), we have the steady-state bequest expression

$$b^c = \frac{R^2}{R^2-n^c}\left\{[(1+g)h(e^c)xR+e^cR]\left(\frac{\theta(c_1^c)}{\theta(n^c)-\theta(c_1^c)}+\frac{1}{R^2}\right)-\frac{\theta(n^c)}{\theta(n^c)-\theta(c_1^c)}\left[h(e^c)+\frac{(1+g)h(e^c)}{R}\right]\right\} \tag{17}$$

We can see that it is the elasticities of the utility and altruism functions that matter. If utility and altruism functions show constant elasticities of substitution, their specific function forms will not influence the consumption and bequests in the steady state.

Plugging the specific functions into (16) and (17), we can get the specific form of I^c and an equation in which bequests are determined.

Appendix 7. The proof of Proposition 7

From Eq. (21) and the budget constraint,

$$c_1^t = \mu_1 I^t, c_2^t = \mu_2 I^t, c_3^t = \mu_3 I^t \tag{18}$$

where μ_1, μ_2, μ_3 are defined as in Proposition 7. $I^t = c_1^t + \frac{c_2^t}{R} + \frac{c_3^t}{R^2}$. The rest of the proof is similar to that of Proposition 6.

Acknowledgement We would like to thank the editor, Alessandro Cigno, and three anonymous reviewers for their important comments. We also want to thank Otávio Bartalotti, Elizabeth Hoffman, Joshua Rosenbloom, Sunanda Roy, Quinn Weninger, John Winters, Wendong Zhang, and participants in the Graduate Students Seminar at Iowa State University for helpful comments. We thank Bryan Jackson for proofreading the manuscript. The authors are responsible for all remaining errors.

Declarations

Conflict of interest The authors declare no competing interests.

References

Adsera A (2006) An economic analysis of the gap between desired and actual fertility: the case of Spain. Rev Econ Household 4:75–95

Bachrach CA, Morgan SP (2013) A cognitive–social model of fertility intentions. Popul Dev Rev 39(3):459–485

Beaujouan E, Berghammer C (2019) The gap between lifetime fertility intentions and completed fertility in Europe and the United States: a cohort approach. Popul Res Policy Rev 38(4):507–535

Becker GS, Barro RJ (1988) A reformulation of the economic theory of fertility. Q J Econ 103(1):1–25

Becker GS, Lewis HG (1973) On the interaction between the quantity and quality of children. J Polit Econ 81(2):S279–S288

Cao D, Werning I (2018) Saving and dissaving with hyperbolic discounting. Econometrica 86(3):805–857

Diamond P, Köszegi B (2003) Quasi-hyperbolic discounting and retirement. J Public Econ 87(9-10):1839–1872

Doepke M, Tertilt M (2018) Women's empowerment, the gender gap in desired fertility, and fertility outcomes in developing countries. AEA Papers and Proceedings 108:358–362

Frederick S, Loewenstein G, O'donoghue T (2002) Time discounting and time preference: a critical review. J Econ Lit 40(2):351–401

Galor O, Zeira J (1993) Income distribution and macroeconomics. Rev Econ Stud 60(1):35–52

Gruber J, Köszegi B (2001) Is addiction "rational"? theory and evidence. Q J Econ 116(4):1261–1303

Günther I, Harttgen K (2016) Desired fertility and number of children born across time and space. Demography 53(1):55–83

Ikeda S, Kang MI, Ohtake F (2010) Hyperbolic discounting, the sign effect, and the body mass index. J Health Econ 29(2):268–284

Jones LE, Schoonbroodt A, Tertilt M (2010) Fertility theory: can they explain the negative fertility-income relationship? In: Shoven J (ed) Demography and the economy. University of Chicago Press, Chicago, pp 43–100

Laibson D (1997) Golden eggs and hyperbolic discounting. Q J Econ 112(2):443–478

Nakagawa M, Oura A, Sugimoto Y (forthcoming) Under- and over-investment in education: the role of locked-in fertility. J Popul Econ

O'Donoghue T, Rabin M (1999) Doing it now or later. Am Econ Rev 89(1):103–124

Phelps ES, Pollak RA (1968) On second-best national saving and game-equilibrium growth. Rev Econ Stud 35(2):185–199

Pritchett (1994) Desired fertility and the impact of population policies. Popul Dev Rev 20(1):1–55

Samuelson PA (1937) A note on measurement of utility. Rev Econ Stud 4(2):155–161

Thomson E, McDonald E, Bumpass LL (1990) Fertility desires and fertility: hers, his, and theirs. Demography 27(4):579–588

van der Pol M, Cairns J (2002) A comparison of the discounted utility model and hyperbolic discounting models in the case of social and private intertemporal preferences for health. J Econ Behav Organ 49(1):79–96

Wang M, Rieger MO, Hens T (2016) How time preferences differ: evidence from 53 countries. J Econ Psychol 52:115–135

Wigniolle B (2013) Fertility in the absence of self-control. Math Soc Sci 66(1):71–86

Wrede M (2011) Hyperbolic discounting and fertility. J Popul Econ 24(3):1053–1070